A solution to the donkey sentence problem

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(The original appeared as "A Solution to the Donkey Sentence Problem" *Analysis* 2015, *75 (4): 554-557.* It contained a mistake, pointed out to me by Thomas Ede Zimmermann. A correction will appear in the journal. This version incorporates the correction, in a way that produces a better formulation of the general idea.)

The problem is that some conditionals use the singular indefinite article in a way that seems both like an existential and a universal quantifier. But it can't be both. In fact it can't be either. For example,

(*) if he has a gun he'll fire it at the police

(So don't let him anywhere near the demo.) This cannot be

$$\exists x\,(Gx \,\&\, Fx)$$

as he may not have a gun. And it cannot be

$$\forall x\,(Gx \supset Fx)$$

as that entails that if he has seven guns he will fire them all at the police. (The puzzle comes from Geach 1962. Geach's examples involve donkeys being beaten. I will not use them, partly because of the donkey-beating but more relevantly because they can easily be read as lacking the crucial features.)

The solution I shall defend is that the quantifier is neither universal nor existential but what I shall call the Geach quantifier, which has features of both. Suppose that we are restricted to two possible guns. Then our sentence says.

$$\forall x\ \forall y(\ \forall z\ (\ Gz \equiv (z=x\ \mathrm{v}\ z=y)\ )\supset (\ Fx\ \mathrm{v}\ Fy\ ))$$

(In effect, if his guns are these two then he'll fire one.)

More generally,with n guns

$$\forall x_1 \ldots \forall x_n\ (\ \forall z\ (Gz \equiv (z=x_1\ \mathrm{v}\ \ldots\ \mathrm{v}\ z=x_n)) \supset (Fx_1\ \mathrm{v} \ldots \mathrm{v}\ \ Fx_n)\ )$$

Note that a singular pronoun with a disjunctive antecedent is common in English, as in "if this was written by John or Marcus, he's a damn hypocrite." Note also that the indefinite article in English can represent quantifiers other than the universal and existential. One example is “exactly one”, as in “an elephant crossed the road, and minutes later a second”. Another is the generic quantifier, as in “a tiger has stripes and long whiskers”, which is not falsified by the existence of albino or shaven tigers. (Schubert & Pelletier 1989 make this point. For a survey of generics see Carlson & Pelletier 1995 and for more recent work see King 2004, 2013. King 2004 invokes a complex contextuality rather than a fixed interpretation as in this paper.)

In full generality the formula would be an instance of a two-parameter generalized quantifier, with l and u as lower and upper bounds for the size of the set of Gs:

$$\forall S\ (\ (l \leq |S| \leq u\ \&\ \forall z\ (z \varepsilon S \equiv Gz)) \supset \exists g\ (g \varepsilon S\ \&\ Fg)\ )$$

For the gun example the appropriate value for l is 1 and for u $\aleph_0$ (1 because it is “a gun” and $\aleph_0$ because I doubt that anyone anywhere has more than denumerably many guns.)

This could be treated as an autonomous two-place quantifier, for example as $\mathscr{G}x\,[Gx, Fx]$.

I have defined the $\mathscr{G}$ quantifier in set-theoretical terms. But we would have to do the same for universal and existential quantifiers if they were defined without using them in their definition. And I am taking this as distinct from both of these, with features that overlap with each. It is widely recognized now that there is a large variety of natural language quantifiers, which we can define given suitable resources. The standard first order pair have some nice features, but we clearly cannot define all others in terms of them. (For the variety of generalized quantifiers in natural language see Peters & Westerståhl 2006, especially chapters 3 and 4. Some nice features of the universal and existential quantifiers are shown by Lindström 1969, which applies when one augments them with others. What analogous results there are when instead of augmenting one replaces these two with others I do not know.)

In defence of this analysis note that the pronoun in the consequent is bound by the quantifier in the antedent, that the conditional form is preserved, and that it is not true in the cases we do not want. Moreover, in the special case of just one gun it entails that that gun is fired, and in the case of more guns it does not entail that they are all fired.

The $\mathcal{G}$(x) analysis reveals continuities with other quantifiers in natural language. The first is with quantifiers that specify not how many but how much. Consider

if she has a dollar more than she needs, she'll give it away.

'It' here is not a particular dollar but a threshold above which her generosity kicks in, just as 'it' in the gun sentence is not a particular gun but a threshold (of 1) above which the shooter's dangerous exhibitionism kicks in.

The second continuity is with some other how-many quantifiers. Examples are

If she has three sons then she'll make two into priests

If he has many guns then he'll fire several of them

If you think long enough about any example you'll find many problems with it.

In all of these a quantifier in the antecedent is linked to one in the consequent, in part because the range of the latter is restricted to that of the former, and in part because the latter amounts to a narrower specification of the former. They could all be defined as quantifiers in their own right, if one had a reason to do so.

There are a number of reactions to the puzzle in the literature They all have problems (See King (2013)). One reaction that is not much discussed, but which arises naturally given my approach, is to take the form as

(EE) $\exists x\, Gx \supset \exists y\, (Gy \,\&\, Fy)$

I shall end by saying briefly why I do not think this is a good solution, though sentences of this form are often true when Geach sentences are. This analysis would make the sentence undesirably true when there are no G, but this might be put down to a quirk of the material conditional. More fundamentally, it forces us to take basic grammatical features of Geach sentences as misleading. What looks like a single quantifier binding a variable is not, and a construction common to many European languages is represented as a quirk of English idiom.

There is also a pragmatic contrast. EE is consistent with

(+) if he has a gun he will not fire it.

(He might have two guns.) But the combination of (*) and "if he has a gun he will not fire it" is hard to evaluate. EE has only the obvious presuppositions that he exists, that there are guns he could have, and that there is an action of firing. But (*) suggests also that firing would be his default action in these circumstances, and that if given two guns he would definitely fire one and not fire the other, then this fact has to be stated explicitly.

I conclude that the generalized quantifier treatment of Geach donkey sentences is at least worthy of further exploration.

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ABSTRACT The problem concerns quantifiers that seem to hover between universal and existential readings. I argue that they are neither, but a different quantifier that has features of each.

KEYWORDS: donkey sentences, Geach, quantifiers