

Suppose, suppose

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when I wrote this paper I was not aware of several relevant articles in the area, in particular Van McGee's two papers: (1989) Conditional probability and compounds of conditionals *Philosophical Review* 98, 485-541. McGee, Vann (1985) Counterexamples to modus ponens *Journal of Philosophy* 82, 462-471

Much of our thinking about counterfactuals is narrative. One tells a story about what could have happened, and successive parts of the story work cumulatively, building up a description of a possible situation without taking back what has been supposed so far. Stories are sequences of sentences. This paper is about a difficulty that occurs when you try to compress a sequence of sentences into a single sentence.

We often express conditional thoughts with a sequence of sentences. Suppose we go abroad next year. Then we wouldn't have to see your uncle. And 'suppose' iterates. Suppose we go abroad next year. And suppose that the car needs replacing. Then we will need a bank loan. There can be as many supposes as you like. As a result we can express very complicated conditional thoughts, using fairly simple syntax, as long as we ignore the pedant's requirement that every sentence must express a self-sufficient proposition. I am interested in the cases where this indefinitely extendible supposing indicates something like the counterfactual conditional. (We normally indicate this by saying e.g. 'Suppose we had gone abroad last year', rather than, say, 'Suppose we did go abroad last year'.)

Suppose that for some reason you want to compress a string of supposes into a single sentence. What would be its logical form? At first sight one might think that Suppose p<sub>1</sub>. ...Suppose p<sub>n</sub>. Then q amounts either to If (p<sub>1</sub> & ... & p<sub>n</sub>) then q. or to if p<sub>1</sub> then (if p<sub>2</sub>...then (if p<sub>n</sub> then q)..). In my 'Double Conditionals' (*Analysis*.1990) I discussed the case where n =2. I argued that many sentences with the surface syntax of if p & q then r are not actually embeddings of a conjunction or a conditional in a conditional. I would now add that many of these can be most naturally expressed in 'suppose' idiom. I also argued, in effect, that neither if p<sub>1</sub> & p<sub>2</sub> then q nor if p<sub>1</sub> then if p<sub>2</sub> then q nor any other iteration of if and and could do the job. And the examples there can easily be extended to cases where n > 2. Extending the notation of that paper, the claim is as follows. There is a conditional "if p<sub>1</sub>/.../p<sub>n</sub> then q" which expresses what "Suppose p<sub>1</sub>....Suppose p<sub>n</sub>. Then q" does. It cannot be defined in terms of *and* and *if*.<sup>1</sup>

If supposes are expressions of narrative thinking this irreducibility is not surprising. Jack says "suppose [P] we go abroad next year." Jill considers that they had planned on an expensive holiday in Japan sometime soon, and that the most likely other expense would be replacing their car, which is ageing but unlikely to break down without warning., and says "And suppose [Q] that the car needs replacing." "Then" replies Jack "[R] we will take out a bank loan." Consider the thought If P/Q then R, i.e. If we go abroad next year / the car needs replacing then we will take out a bank loan. It is not

If (P&Q) then R

with the Lewis-Stalnaker if. For in the nearest world in which they go abroad and the car needs replacing is one in which they have enough warning that it is about to pack up and so go to France rather than Japan. And it is not

If P then (if Q then R)

because in the nearest world in which they go abroad it is to Japan and the nearest world to that in which the car needs replacing is one in which they, knowing the car needs replacing, don't go to Japan and so don't need a loan. And it is not

If P then (if P & Q then R)

for similar reasons: the nearest world to their Japanese holiday world in which P & Q is true is one in which they anticipate the car's demise and go to France instead.

The example makes it clear why repeated supposes don't collapse into and and if. For supposing P & Q in a story is quite different from first supposing P and then supposing Q. It all comes down to the fact that repeated supposes are cumulative. Having set it up so P happened one way, you don't shift the understanding of the story-so-far. This can be put slightly more carefully and generally, though not really more precisely, as a definition. If  $p_1/\dots/p_n$  then q is true when q is made true by the smallest departure from actuality that would make p true and would make  $p_1$  true while preserving the reasons for which p is made true, ... and which would make  $p_n$  true while preserving the reasons for which  $p_1\dots p_{n-1}$  are made true. It is easy to see why this is a conditional idea we have much use for, and very plausible that it does not reduce to a combination of if and and.

Can this be put more formally, without talk of 'reasons for which' a proposition is true in a world? I think so. I am not at all sure that the underlying idea is made any clearer thereby. But a formal treatment does at any rate show its logical coherence.

We need to be able to say that p is true in world v and in world w, and holds in w for the same reasons that it holds in v. That amounts to saying that w does not differ from v in one particular respect. So in a many-dimensional polyverse in which similarity in that respect is measured by, say, horizontal separation, u and v are on a vertical line. (Or more generally, in a subspace of points which do not differ in that respect.) This can, surprisingly, be expressed in terms of a qualitative relation of similarity between worlds. In fact the relation that is needed is  $S(u,v,w)$  'v is at least as similar to u as w is', interpreted so that if v and w are incomparable in similarity to u the assertion is false. In terms of this we can define a relation  $R(u,v,w)$  which holds when w is in a subspace of the set of possible worlds orthogonal to the line joining u and v. ( $R(u,v,w)$  holds when  $(t)((R(v,u,t) \& R(v,t,u)) \supset (R(w,u,t) \& R(w,t,u)))$ .) Then we can define a multigrade subjunctive conditional, that is, one whose antecedent can contain any finite number of propositions, as follows.

If  $P_1/P_2/\dots/P_n$  then R is true when any chain of worlds  $w_1,\dots,w_n$  such that for each i  $w_i$  is the nearest  $(P_1\&\dots\&P_i)$ -world to  $w_{i-1}$  and  $R(@,w_{i-1},w_i)$ , is such that R holds in  $w_n$ .

The n-antecedent case does not in general reduce to a n-1-antecedent conditional. (Though in many particular models irreducible n-antecedent conditionals will be impossible above a given n.) And the 2-antecedent case is irreducible to the ordinary 1-antecedent *if* in this sense: the 1-antecedent counterfactual can be expressed in terms of the similarity relation  $T(u,v,w)$  'v is not more dissimilar from u than w is', interpreted so that if v and w are incomparable in similarity to u the assertion is true, but the 2-antecedent case requires the stronger relation S, defined above, which cannot be defined in terms of T.

The contrast between the strong and the weak similarity relation between worlds may perhaps be of more general use. The device of taking the polyverse to have straight line directions is less likely to be of use in itself, though the formalization of the idea of the reason why a sentence holds in a world may connect with the analysis of causal ideas. (Especially since it involves 'non-backtracking' conditions.) What seems to me most promising is the existence of a formal semantics for a multigrade propositional connective.<sup>2</sup> One reason for finding this interesting is its connection with the narrative roots of commonsense modality, and with the way that our assertions are made against a background of assumptions that more often grow than shrink.

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#### Footnotes

1 The definition in that paper was wrong. It had the unintended consequence that if  $p/q$  then  $r$  is equivalent to if  $p$  then (if  $p \& q$  then  $r$ ). One purpose of this paper is to correct that mistake. At the Analysis 50 conference Jonathan Lowe pointed out my mistake to me, and Jonathan Bennett, Ian Hinkfuss, and Timothy Williamson gave me clues about how to fix it. A good discussion of the usefulness of 'suppose' idioms, but focussing on argument-presenting rather than conditional-asserting uses, is Fisher [1] The editor of Analysis suggested much better examples than the ones I was using.

2 Multigrade relations (but not multigrade connectives) are discussed in Grandy [2], Lewis [3] and Morton [4]. Taylor [6] gives a good formal treatment. The importance of taking the similarity relation to be a partial ordering is emphasised in Pollock [5]. For more on 3-termed similarity relations see Williamson [7].

[1] Alec Fisher 'Suppositions in Argumentation', *Argumentation* 3, (1989) 401-413.

[2] R E Grandy *Advanced Logic for Applications* (Dordrecht: Reidel, 1977).

[3] David Lewis *Parts of Classes* (Oxford: Oxford University Press, 1991).

[4] Adam Morton 'Complex individuals and Multigrade Relations', *Nous* 9, (1975) 309-318,

[5] John Pollock *The Foundations of Philosophical Semantics*, (Princeton: Princeton University Press, 1984).

- [6] Barry Taylor 'Articulated Predication and Truth-Theory', Melbourne Philosophy Preprint 2/82 (1982)
- [7] Timothy Williamson 'First-Order Logics for Comparative Similarity', Notre Dame Journal of Formal Logic 29, (1988) 457-481.